Constraining CFTs

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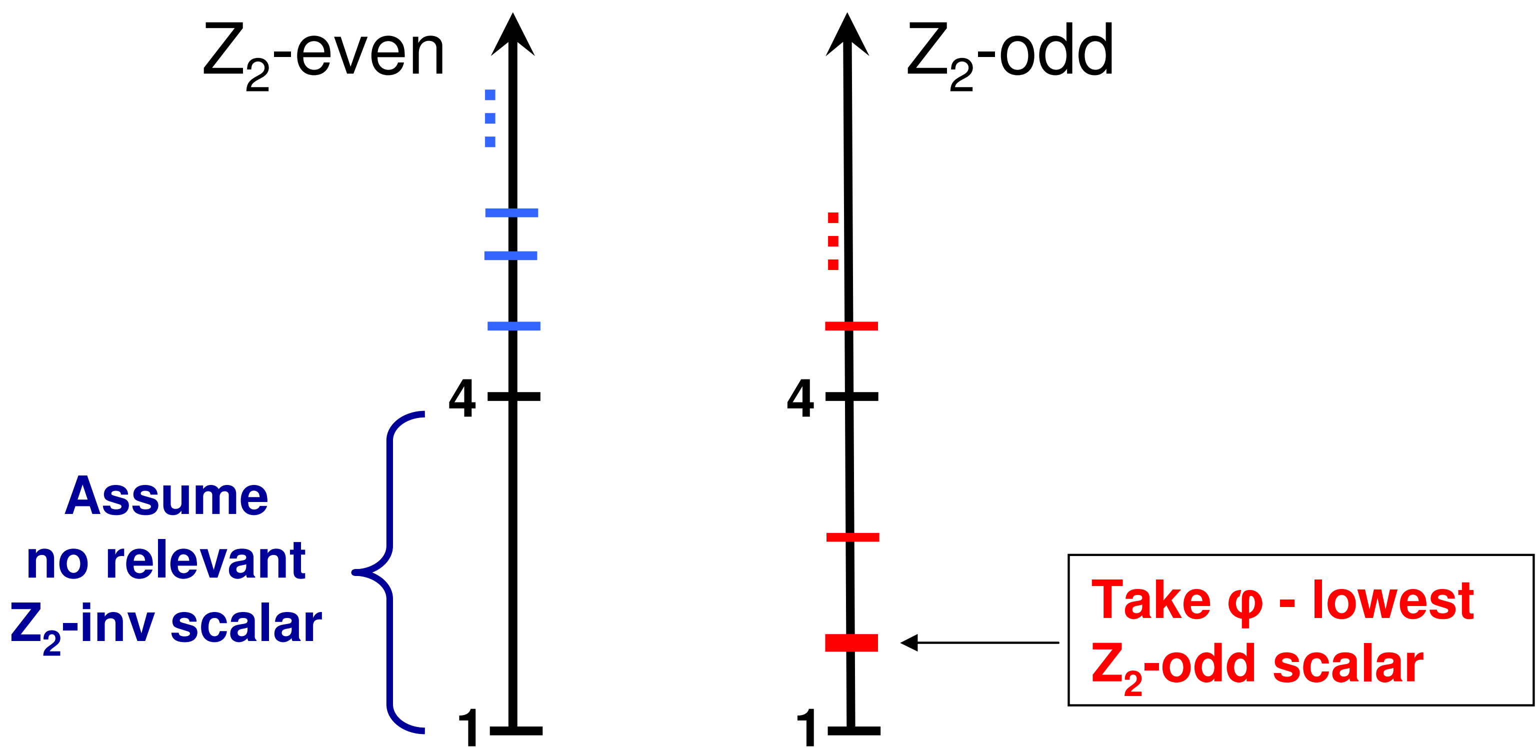
- with **R.Rattazzi, E.Tonni, A.Vichi** 0807.0004
- with **A.Vichi** 0905.2211
- with **F. Caracciolo** 0912.2726

Toy Problem

- Unitary CFT in D=4 + Z_2 symmetry

Scalar spectrum:



Question: What is minimal possible dim(φ)?

(assuming no relevant Z_2-inv scalar)

Why expect dim(φ) $\to$1 is impossible

Consider OPE: $\varphi \times \varphi \supset$ "φ^2" - Z_2-even

As $\dim(\varphi) \to 1$ expect φ 'approaches' free field, and:

$$\dim(\text{“}\varphi^2\text{”}) \to 2$$

- becomes relevant

How to make this argument rigorous?

Classic theorem that **dim(φ)=1 field is free** does not help;
Standard proof uses $\partial^2\varphi=0$; Does NOT generalize to **dim(φ)=1+ε**

Real problem

Consider a QCD-like theory: $\mathcal{L} = \mathbf{Tr}\, F_{\mu\nu}^2 + \bar{\Psi} D_\mu \Psi$

As. free for $N_f < 5.5 N_c$

Expect 'conformal window' for $N_f \to 5.5 N_c$

At the IR fixed point:

- Global symmetry group $G = SU(N_f)_L \times SU(N_f)_R$
- **No G-invariant relevant scalar**

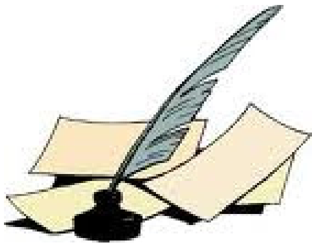
Spectrum of operator dimensions? E.g. $\mathbf{dim}\text{“}\bar{\Psi}\Psi\text{”} = ?$

Why care:

Such operators could play a role of 'composite' Higgs field in Technicolor-like UV-completions of the Standard Model.

$\mathbf{dim}\text{“}\bar{\Psi}\Psi\text{”} \to 1$ would be best. **Lower bound?**

Literature



Rattazzi, V.R., Tonni, Vichi 0807.004, ***V.R., Vichi*** 0905.2211
– solved the toy problem (real problem – work in progress)
V.R., Caracciolo 0912.2726 $\varphi \times \varphi \supset c_{OPE} O, \quad \max c_{OPE} = ?$

Hellerman 0902.2790,
Hellerman, Schmidt-Colinet 1007.0756

similar problems in 2D using modular invariance

Höhn math/0701626
Gaberdiel, Gukov, Keller, Moore, Ooguri 0805.4216

2D under extra assumptions (SUSY, holo factorization)

Douglas 1005.2779 'Spaces of conformal field theories'

Heemskerk, Penedones, Polchinski, Sully 0907.0151
Heemskerk, Sully 1006.0976

'Holography from CFT'

Preliminary idea

2-point
3-point $\Rightarrow$ CFT kinematics

CFT *dynamics* begins at 4-point

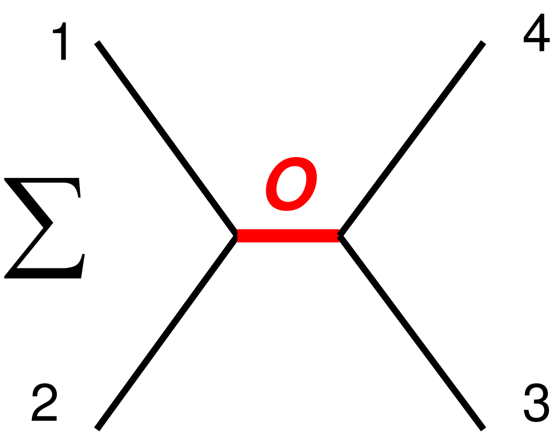
What goes wrong with

$< \phi\phi\phi\phi >$

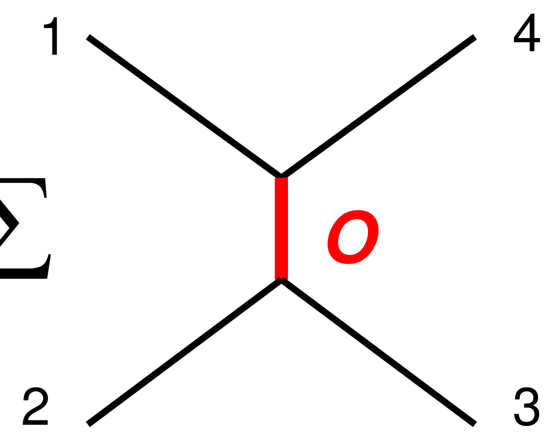
when dim $\varphi \to 1$ but dim(φ^2)>4 ?

Crossing symmetry

$$< \phi\phi\phi\phi > = \sum$$



$$= \sum$$

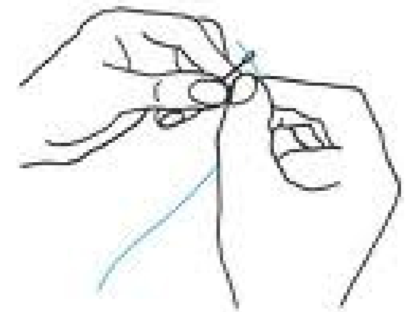


'Bootstrap equation'

Polyakov 1974
Belavin Polyakov Zamolodchikov 1984

Can pull out something?

Preparation 1: **OPE**



$$\varphi(x)\varphi(0) \sim \frac{1}{|x|^{2d}} \sum c_{\Delta,l} \, |x|^{\Delta} \, O_{\Delta,l}(0) + \textbf{descendants}$$

♥ Bose symmetry $\Rightarrow$ Even spins $l = 0, 2, 4...$

♥ Unitarity $\Rightarrow$ 1) real OPE coefficients $c_{\Delta,l} \in \mathbf{R}$

2) lower bounds on operator dimensions:

$$\Delta \geq 1 \qquad (l = 0)$$
$$\Delta \geq l + 2 \qquad (l = 2, 4, 6...)$$

Ferrara, Gatto, Grillo 1974
Mack 1977

♥ No relevant scalar (by assumption): $\Delta \geq 4 \qquad (l = 0)$

BUT: not immediately useful for imposing crossing symmetry

Preparation 2:
Conformal Block Decomposition

$$<\phi_1\phi_2\phi_3\phi_4> = \frac{G(u,v)}{(x_{12})^{2d}(x_{34})^{2d}}$$

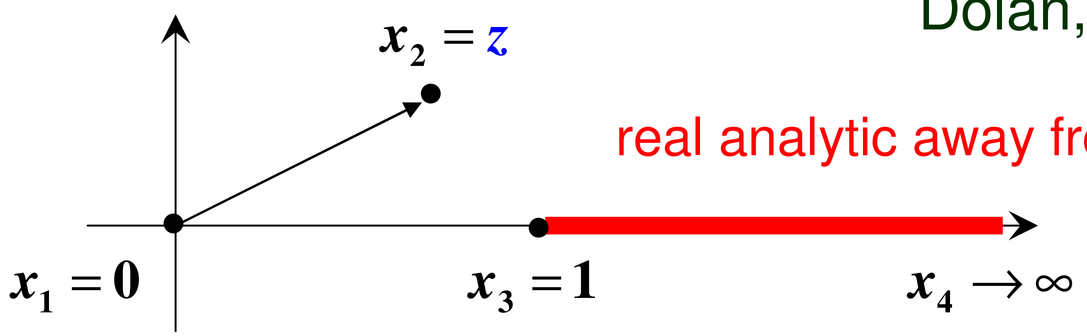
$$d = \dim \phi$$

$$G(u,v) = \sum (c_{\Delta,l})^2 \, \mathbf{CB}_{\Delta,l}(u,v)$$

$$\mathbf{CB}_{\Delta,l}(0,z,1,\infty) = \frac{k_{\Delta+l}(z)k_{\Delta-l-2}(\bar{z}) - (z \leftrightarrow \bar{z})}{z-\bar{z}}$$

$$k_\beta(z) = z^{\beta/2+1}{}_2F_1(\beta/2,\beta/2,\beta,z)$$

Dolan, Osborn 2001



real analytic away from the cut

(Obtained by a) summing the OPE power series
or b) as spherical harmonics of the conformal group + OPE boundary conditions)

Crossing + CB = Sum rule

$$v^d G(u,v) = u^d G(v,u)$$

$$G(u,v) = 1 + \sum (c_{\Delta,l})^2 \,\mathbf{CB}_{\Delta,l}(u,v)$$

Sum rule:

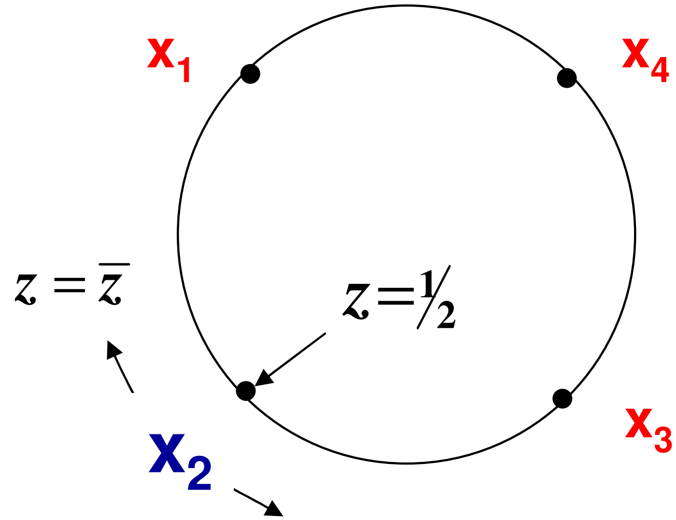
$$1 = \sum (c_{\Delta,l})^2 F_{d,\Delta,l}(u,v)$$

$$F = \frac{v^d \,\mathbf{CB}_{\Delta,l}(u,v) - u^d \,\mathbf{CB}_{\Delta,l}(v,u)}{u^d - v^d}$$

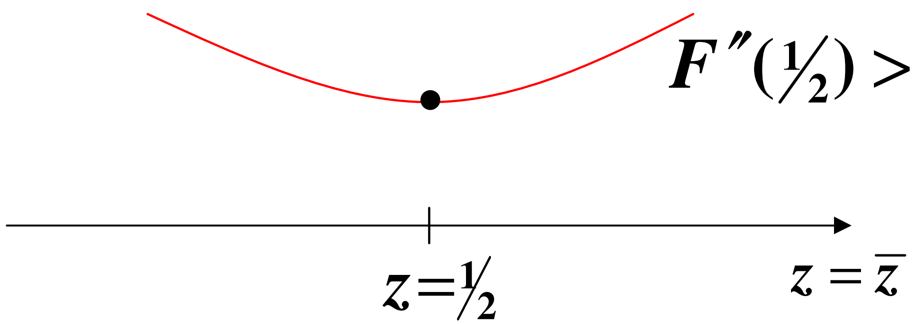
Functional equation involving **squares** of OPE coefficients

Solving toy problem

Vary x_2 near z=1/2
(4 points in vertices of a square)



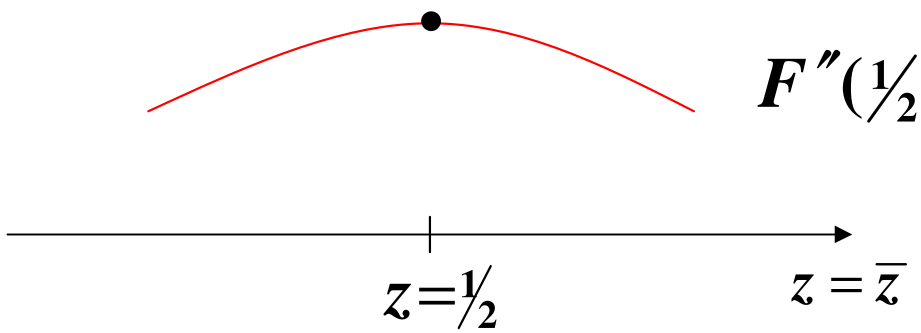
Fix d~1 and study behavior of different terms:



$F''(½) > 0$ for all

$\Delta \geq 4$	$(l=0)$
$\Delta \geq l+2$	$(l=2,4,6...)$

$\Rightarrow$ sum rule $1=\sum c^2_{\Delta,l} F_{d,\Delta,l}$ has no solutions

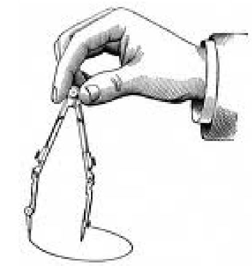


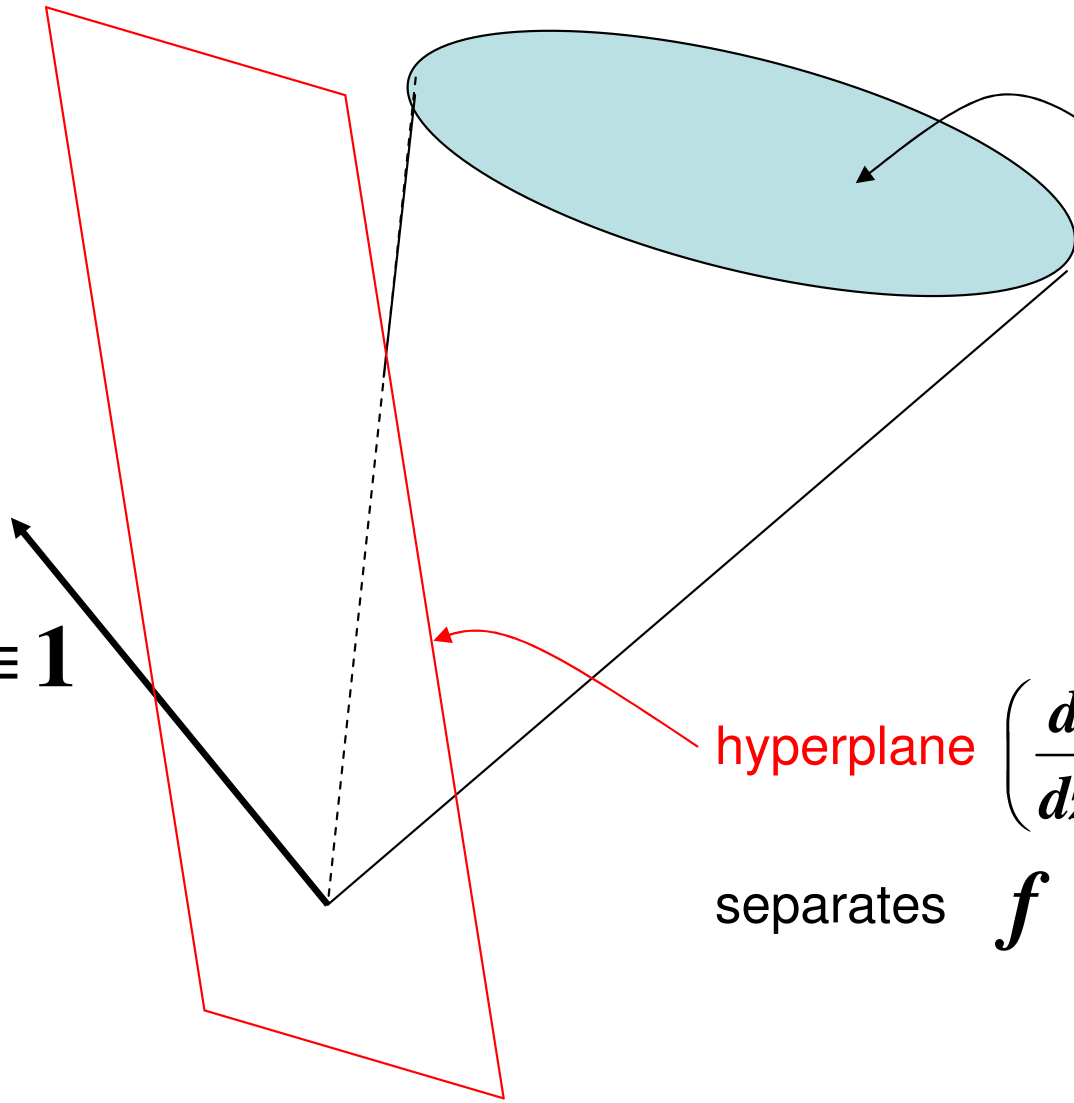
$F''(\tfrac{1}{2}) < 0$

$\Delta = 2, l = 0$

$\Rightarrow$ free scalar theory may exist

Sum rule: Geometric interpretation





$f \equiv 1$

convex cone generated by $F_{d,\Delta,l}(u,v)$

hyperplane $\left(\frac{d}{dz}\right)^2 f = 0$

separates $f \equiv 1$ from the cone

More general linear combinations useful? $\sum \lambda_{m,n} \left(\frac{d}{dz}\right)^m \left(\frac{d}{d\bar{z}}\right)^n$

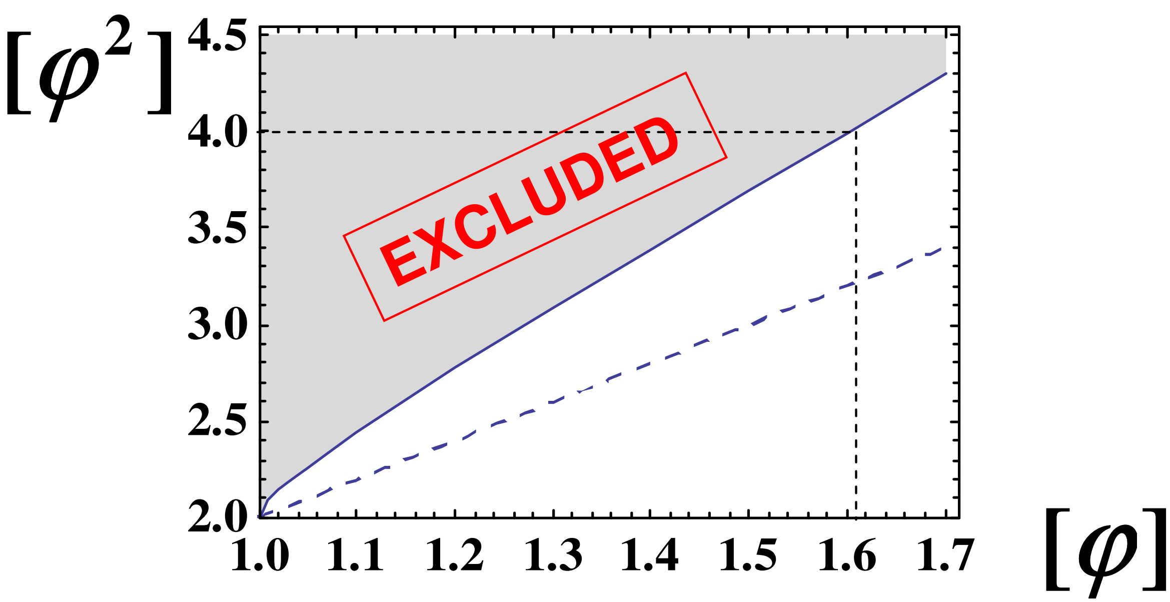
Generalization

Free field theory limit approached continuously:

$$\mathbf{dim}(\varphi) \to 1 \quad \Rightarrow \quad \mathbf{dim}(\text{“}\varphi^2\text{”}) \to 2$$

More precisely, there is a numerical bound:

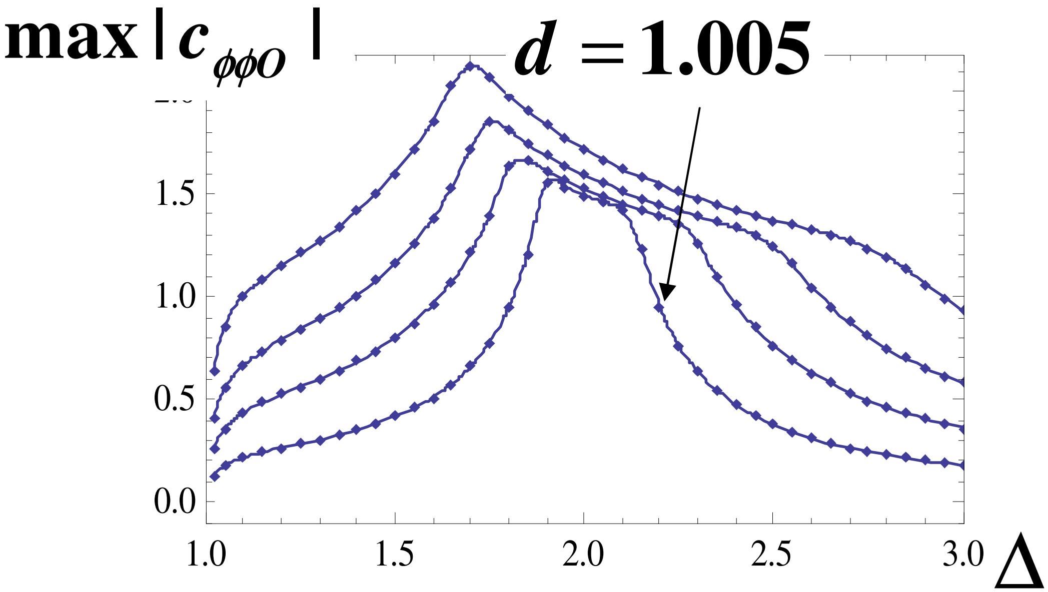
$$\mathbf{dim}(\text{“}\varphi^2\text{”}) \leq f(\mathbf{dim}\,\varphi)$$

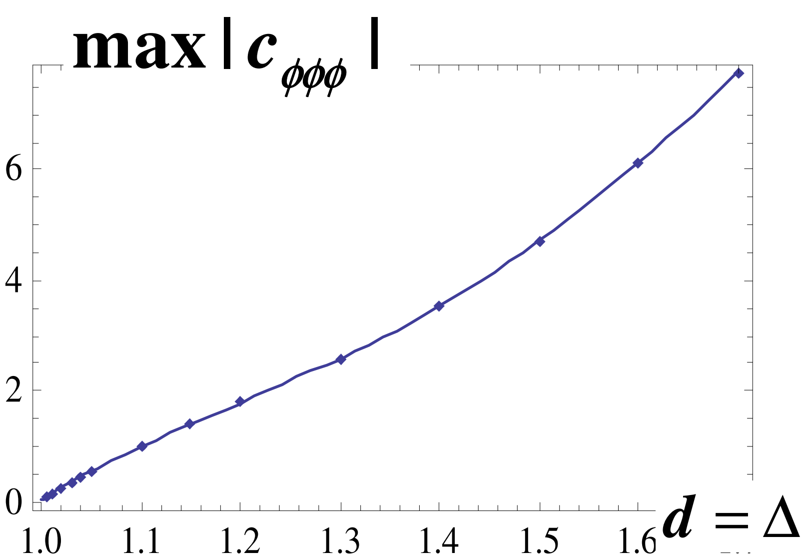


Another application

$$\phi_d \times \phi_d \supset c_{\phi\phi O} \cdot O_\Delta$$

$$\max | c_{\phi\phi O} | = f(d, \Delta) = ?$$

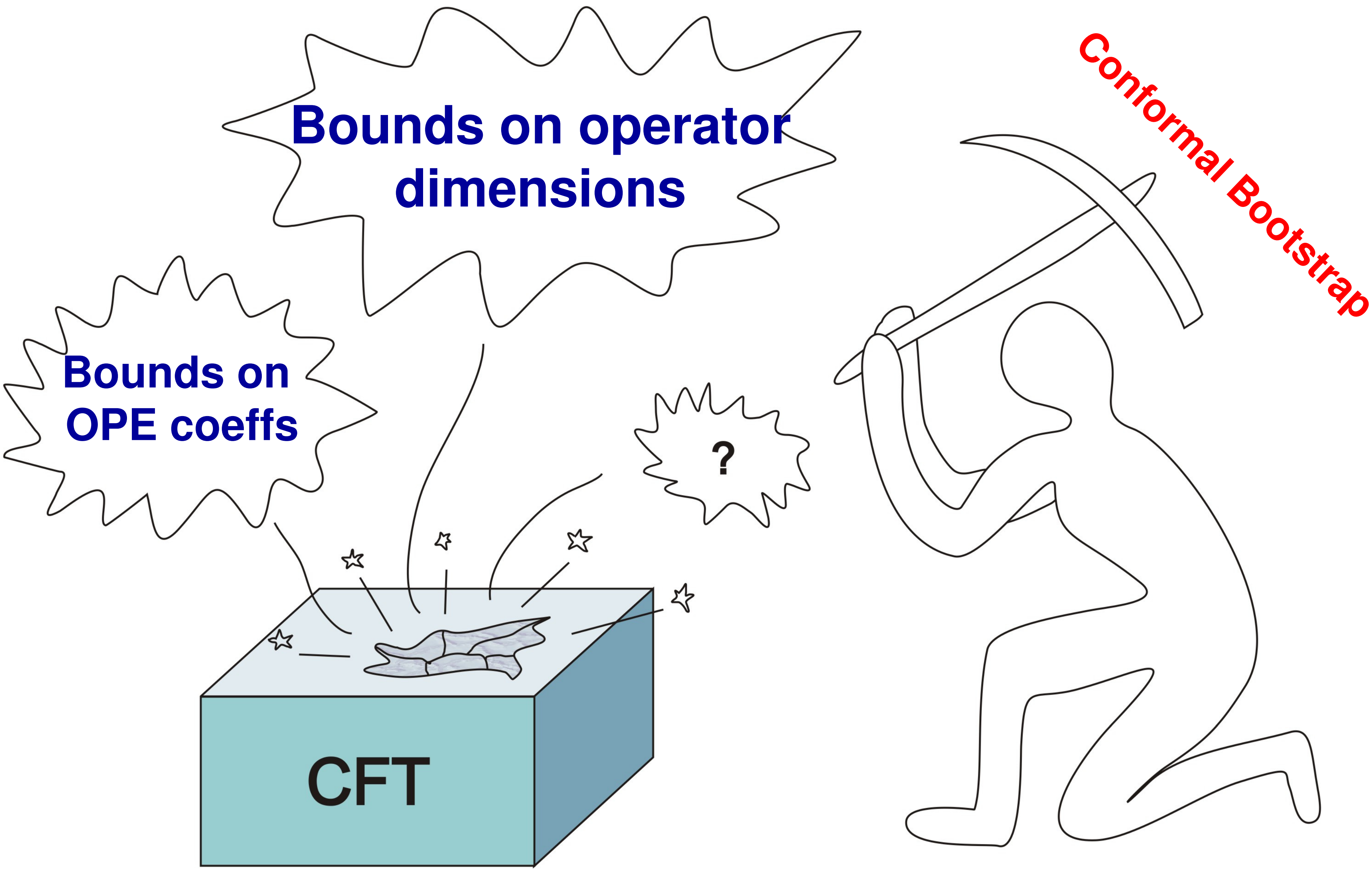




► ‘Rigorous limits on the <u>interaction strength</u> in CFT’

► Important for unparticle phenomenology

Conclusions



BACKUP

2D and 3D examples

show that $\gamma_{\phi^2} >> \gamma_{\phi}$ is not impossible.

Ising model: $\sigma \times \sigma = \mathbf{1} + \varepsilon$

2-dimensions (Onsager)	$[\sigma] = \mathbf{1/8}, \quad [\varepsilon] = \mathbf{1}$
3-dimensions (ϵ- and high-T expansions, Monte-Carlo)	$\gamma_{\sigma} \approx \mathbf{0.02}, \quad \gamma_{\varepsilon} \approx \mathbf{0.4}$

Extending analysis to 3d?

difficulty: finding 3d conformal blocks

(in odd dim's conformal blocks do not factorize as f(z)f(zbar))

Non-trivial extension for globally-symmetric case?

$$\phi_a \times \phi_b = \delta_{ab}(1 + O^{(1)}) + O^{(2)}{}_{ab} + \ldots$$

$$\ldots \supset J^{\mu}{}_{ab}$$

-two inequivalent crossing-symmetric 4-pt functions:

$$\langle \phi_1 \phi_1 \phi_1 \phi_1 \rangle \quad \langle \phi_1 \phi_2 \phi_1 \phi_2 \rangle$$

-OPE contains singlets and symmetric-traceless tensors (*even spin*);

antisymmetric tensors (*odd spin*)

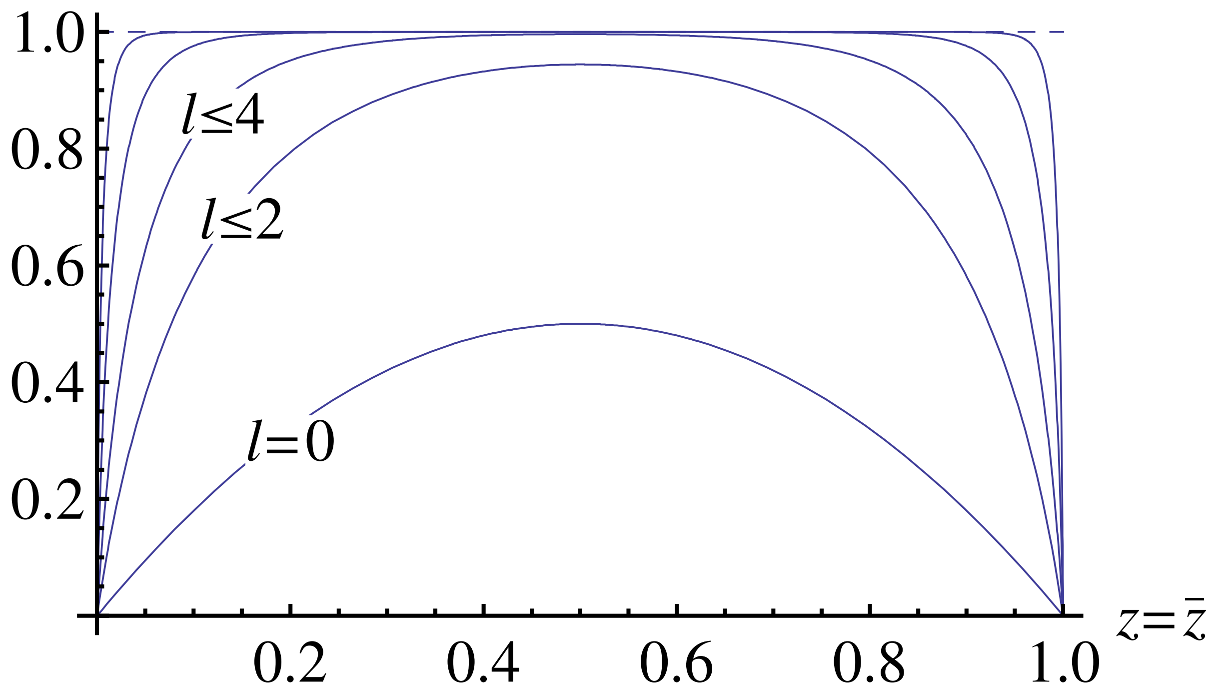
Can one bound $[O^{(1)}]$ in a model-independent way?

Sum rule convergence in free scalar theory

$$\phi\times\phi=\sum_{l=2n}\phi\,\overleftrightarrow{\partial}^{2n}\phi$$

twist 2 fields only

$$\lambda_l^2=2^{l+1}\frac{(l!)^2}{(2l!)^2}$$



Monotonic convergence